Factor Analysis and Correspondence Analysis Composite and Indicator Scores of Likert Scale Survey Data

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Background and Objectives

"Individual survey questions are often imperfect measures of the population traits of interest and there is frequently a need to distill survey data down into relevant information about the population or populations."

- Fricker et al (2012)

Surveys are valuable tools in complex irregular warfare arenas for gaining insight into population attitudes, beliefs, and perceptions. Factor Analysis and Correspondence Analysis unlock statistical power underlying survey response data.

Objectives

- Why composite scores?
- Demonstration: Controlled experiment
- Scoring comparisons
- Breaking the model
- Advantages/disadvantages
- How does this analysis help the Alliance think differently?
- Questions

Why Composite/Indicator Scores?

> *"Occasionally, we are confronted with a client who wants to simply average each participant's response values on several questions to arrive at a composite score for the domain which the questions are believed to be assessing. This is generally a bad idea because, it treats each question as contributing to the composite score equally – which is often not the case when one considers the latent variable structure of what one is attempting to measure or assess."*
>
> - Starkweather (2012)

Factor Analysis Calculation (4 steps)[1]

- Linearly related data
- 1: Recode Likert responses to numeric responses
- 2: Factor analysis→ data structure
- 3: Factor scores and loadings
- 4: Rescale factor scores to reflect original semantic
  - Factor loadings
  - Weighted mean (original data)
  - Weighted standard deviation (original data)

Correspondence Analysis Calculation (4 steps)

- Data need not be linearly related
- 1: Construct contingency table
- 2: Correspondence analysis→ data structure (biplot)
- 3: Perpendicular projection
  - Column coordinate points→ Row coordinate vectors
  - Point-intercept distances
- 4: Rescale indicator scores to reflect original semantic
  - Likert response distances
  - Visualize varying distances between scales

Controlled Experiment

	requirement1	requirement2	requirement3	requirement4	requirement5	requirement6	requirement7	requirement8	requirement9
1	6	2	4	5	4	4	7	7	7
2	5	2	4	4	4	3	1	4	5
3	6	2	4	7	5	4	7	7	7
4	5	2	4	1	3	2	4	5	7
5	1	1	3	5	4	4	3	5	6
6	1	1	3	6	5	4	7	6	7
7	4	1	4	7	5	4	7	7	7
8	1	1	3	4	4	3	2	4	5
9	3	1	3	3	4	3	6	6	7
10	6	2	4	5	4	4	7	7	7
11	3	1	3	2	3	2	4	5	7
12	4	1	4	7	5	5	3	5	6
13	1	1	3	6	4	4	4	5	7
14	7	2	4	1	3	2	7	7	7
15	1	1	3	2	3	2	1	4	5
16	6	2	4	4	4	3	2	4	6
17	7	2	4	3	4	3	4	5	7
18	3	1	3	2	3	2	2	4	6
19	7	2	4	5	4	4	7	6	7
20	6	2	4	6	5	4	3	6	7
21	4	1	4	3	4	3	7	6	7
22	5	2	4	1	3	2	6	6	7
23	3	1	3	4	4	3	1	4	5
24	6	2	4	6	4	4	4	5	7
25	4	1	3	5	4	4	2	4	5
26	5	2	4	3	4	3	6	6	7
27	7	2	4	5	4	4	5	6	7
28	3	1	3	4	4	3	3	5	6
29	3	1	3	4	4	3	4	5	7
30	3	1	3	4	4	3	6	6	7
31	6	2	4	1	3	2	7	6	7
32	2	1	3	5	4	4	7	7	7
33	1	1	3	4	4	3	3	5	6
34	2	1	3	5	4	4	4	5	7
35	7	2	4	3	3	3	7	6	7
36	4	2	3	1	3	2	5	6	7
37	7	2	4	4	4	3	7	7	7
38	7	2	5	7	5	5	2	4	6
39	4	2	3	2	3	2	3	5	6
40	6	2	4	3	3	3	2	4	6

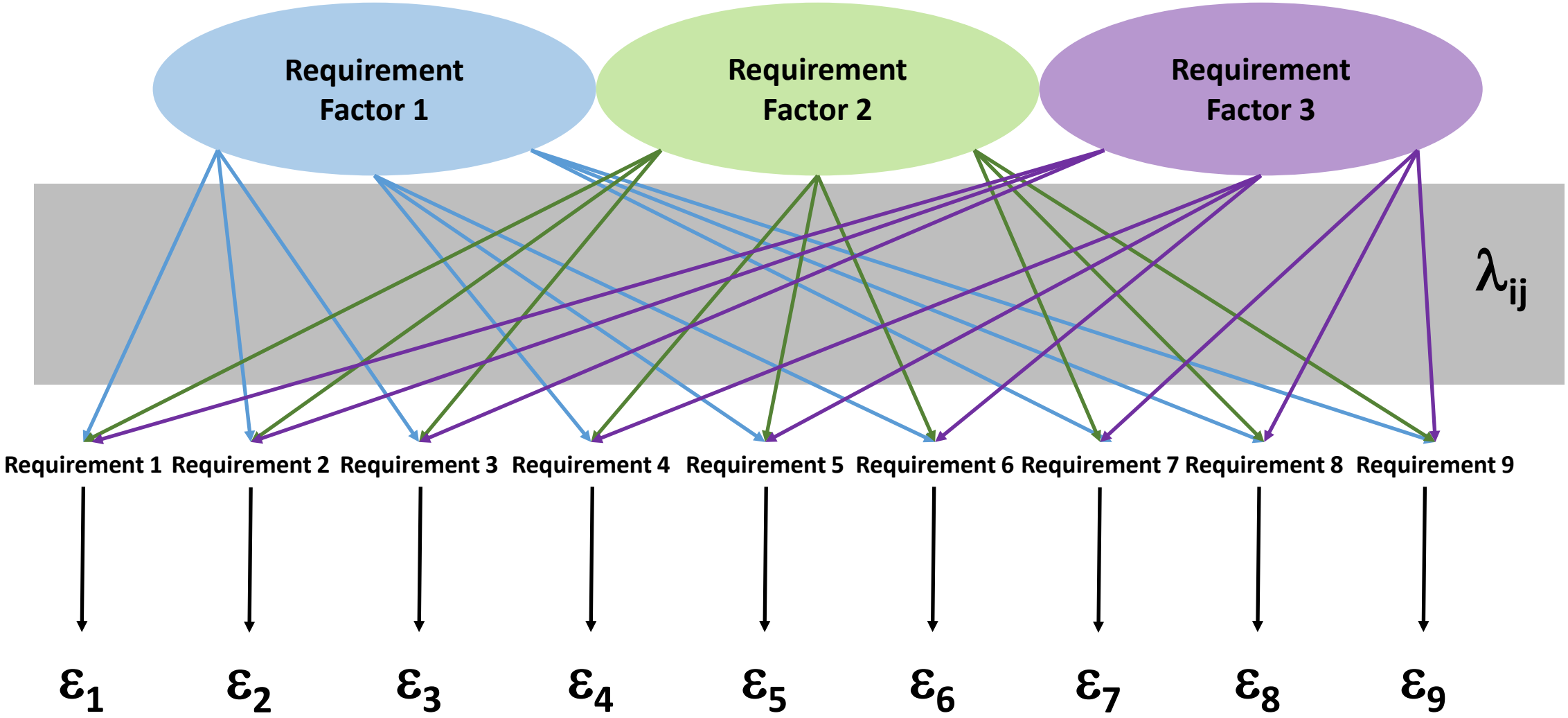
Step 1 (Factor Analysis): Recode

Contingency Table

	X1	X2	X3	X4	X5	X6	X7
requirement1	6	2	7	6	4	8	7
requirement2	19	21	0	0	0	0	0
requirement3	0	0	18	21	1	0	0
requirement4	5	4	6	9	8	4	4
requirement5	0	0	11	23	6	0	0
requirement6	0	9	15	14	2	0	0
requirement7	3	6	6	7	2	4	12
requirement8	0	0	0	9	13	11	7
requirement9	0	0	0	0	5	10	25

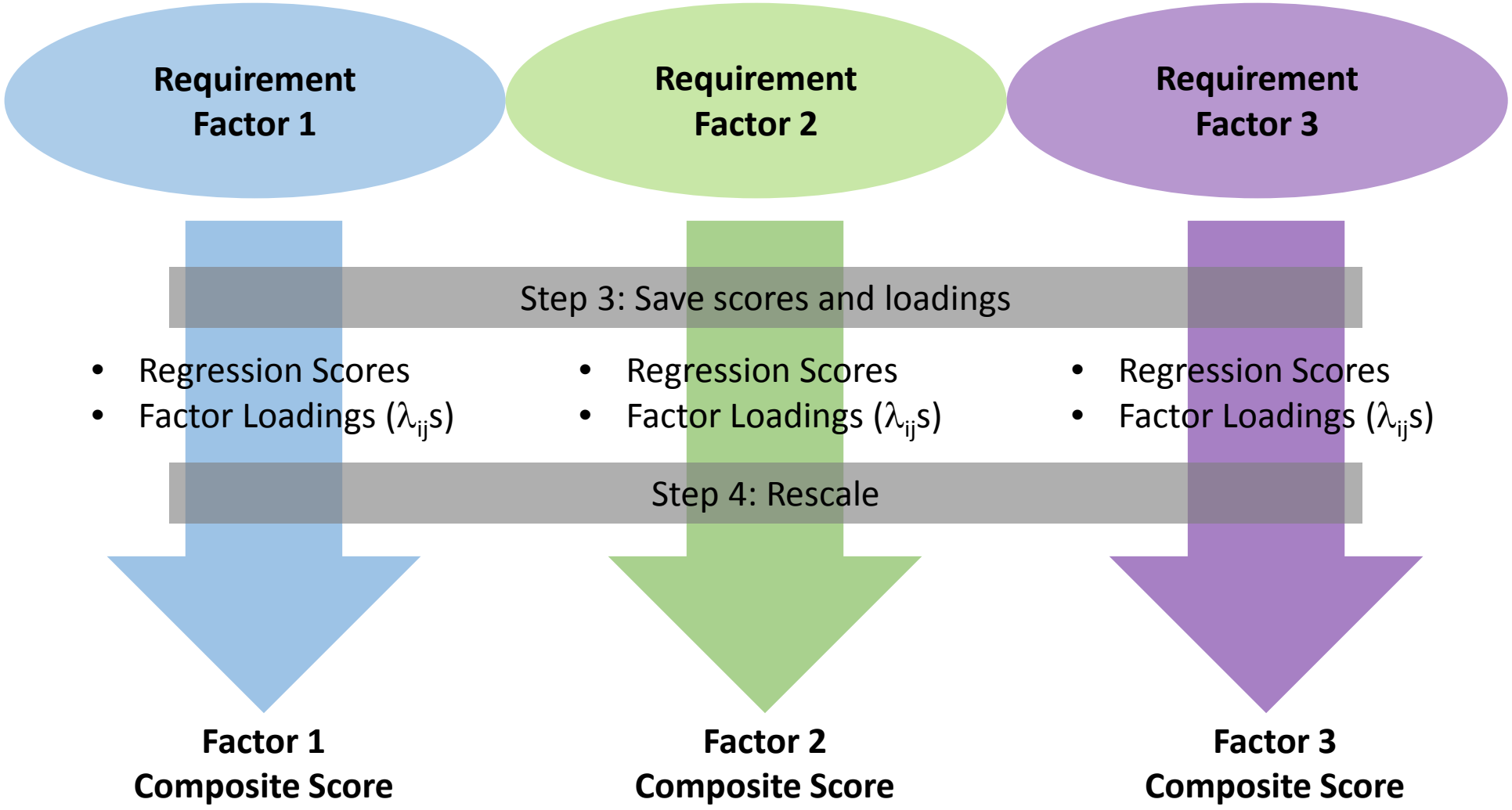
Step 1 (Correspondence Analysis): Contingency table

Factor Analysis of Requirement Variables



Step 2: Factor Analysis

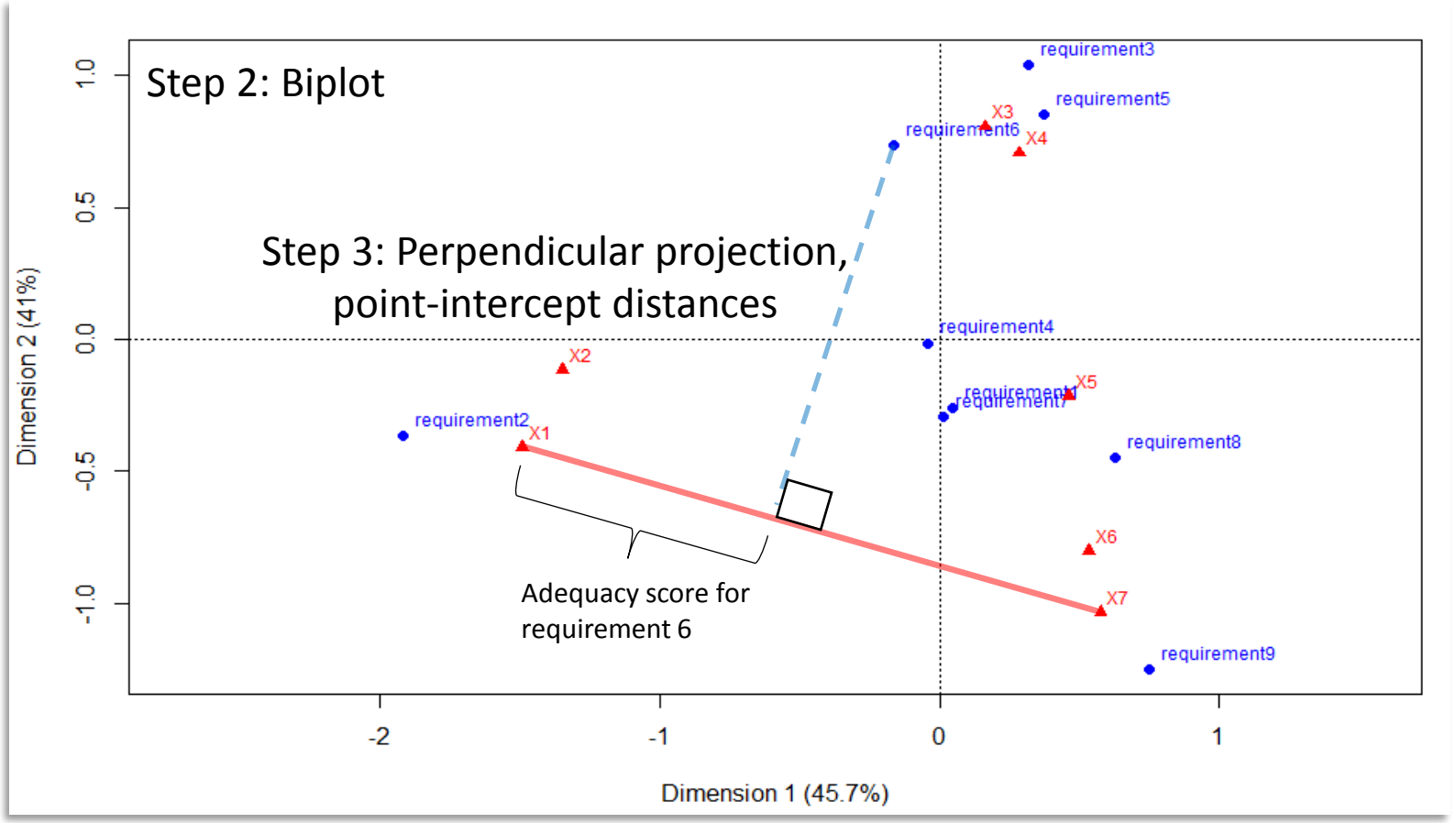
Factor Analysis Composite Scores



Correspondence Analysis Projection Scores

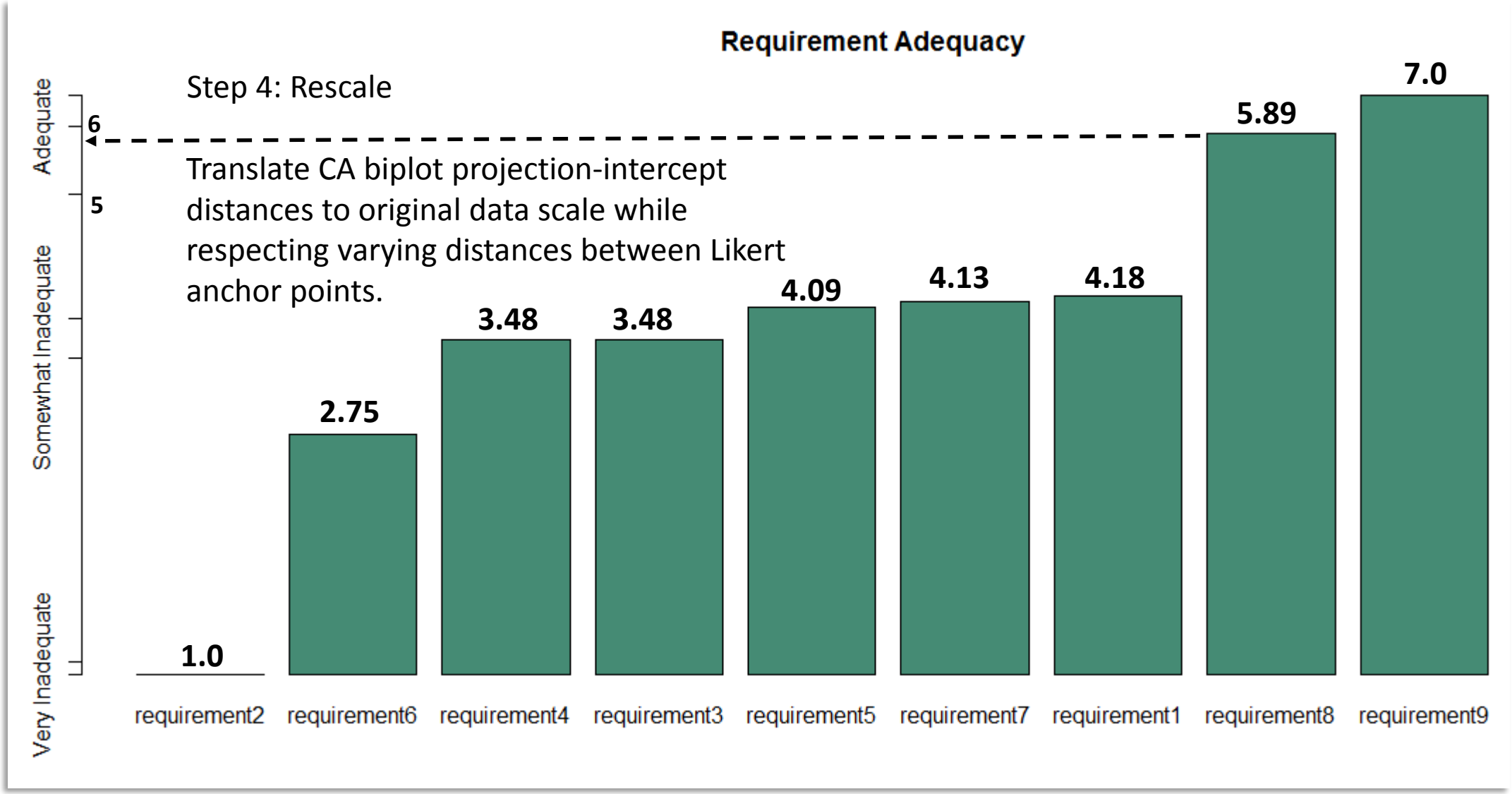
"In Correspondence Analysis only distances between the row points or between column points can be interpreted directly. The relation between row and column points can only be assessed by projection."

- Borg and Groenen (2005)

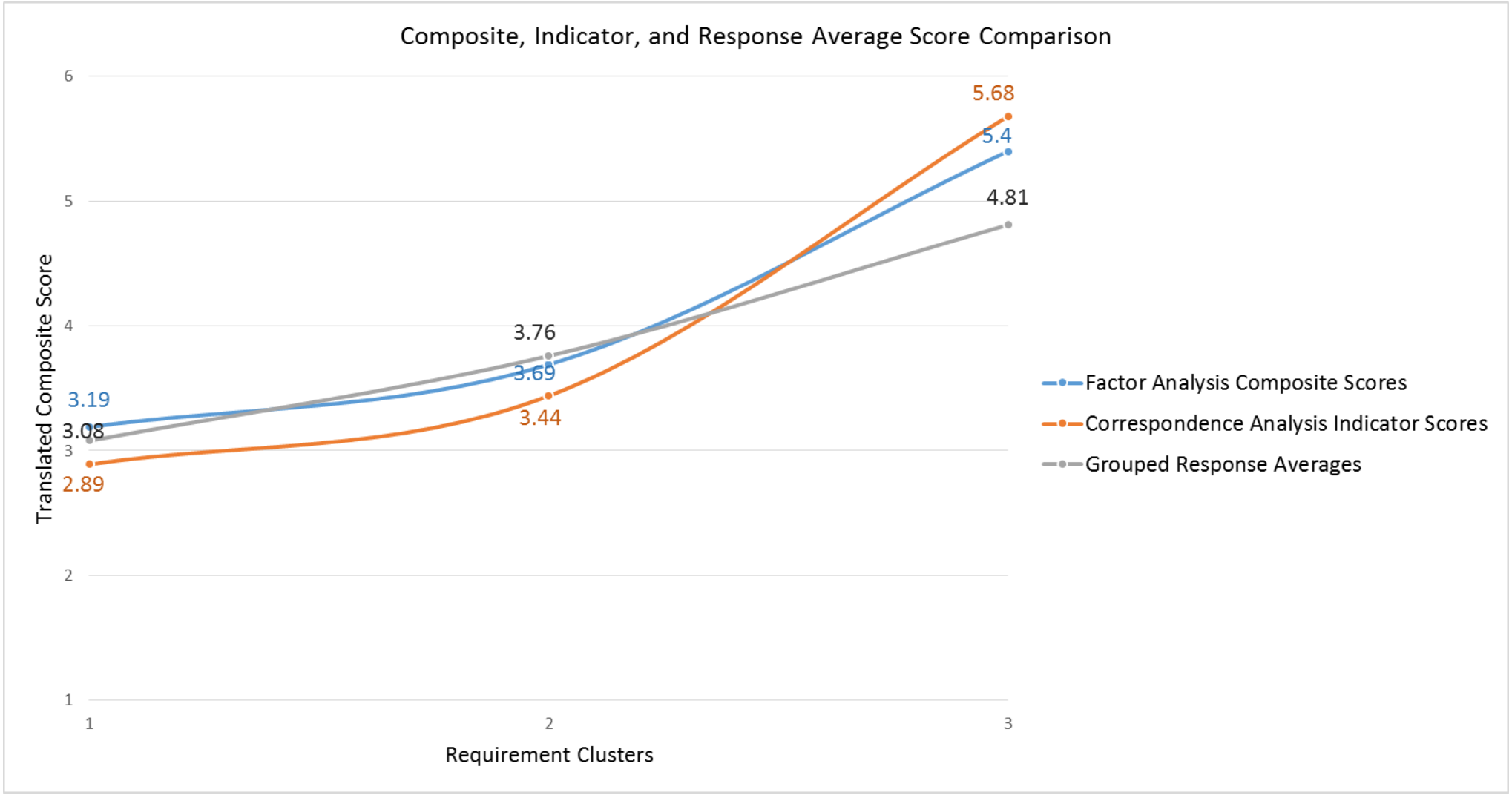


Translated Correspondence Analysis Projection Scores

Varying "distance" between Likert scale anchor points.

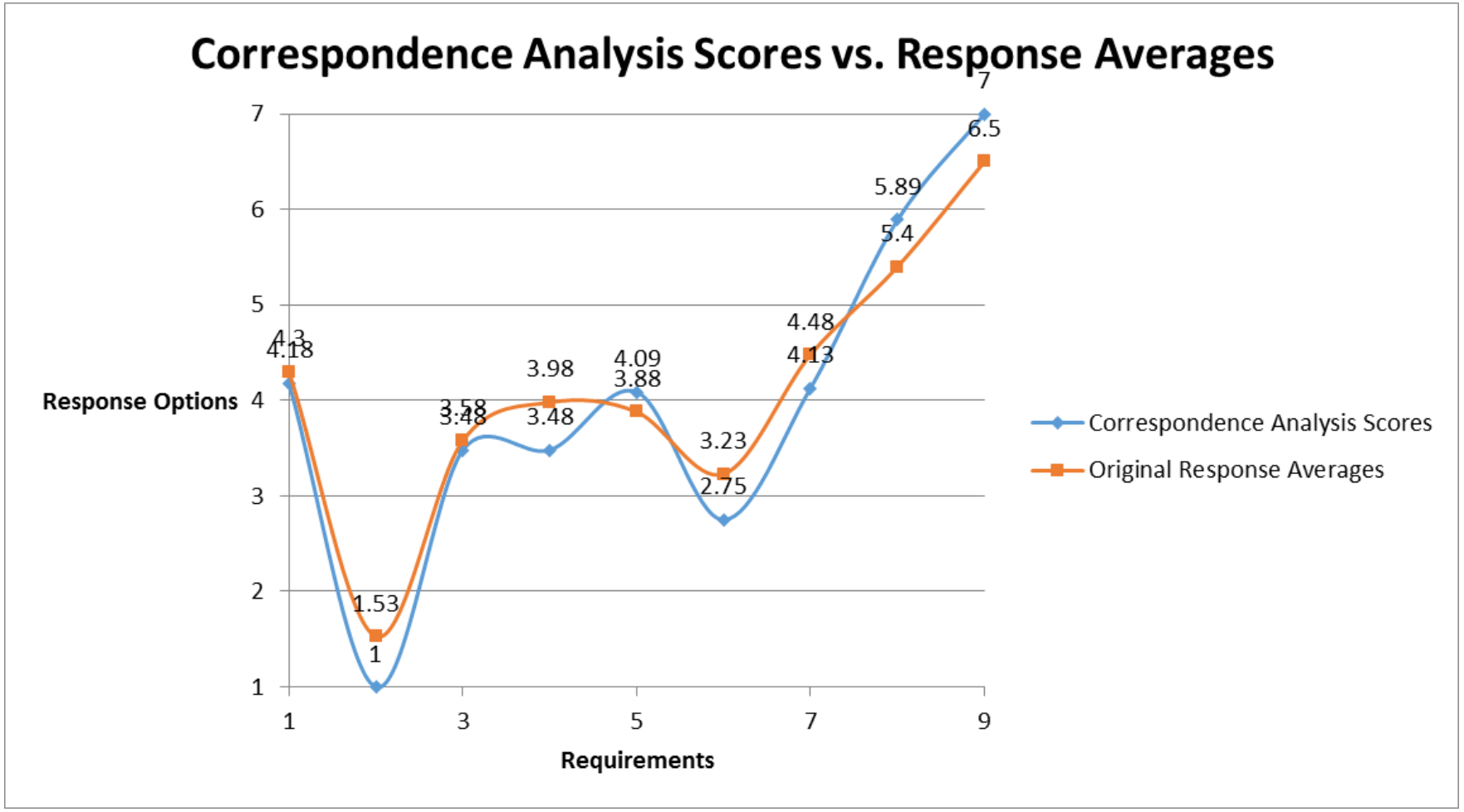


Factor and Correspondence Analysis Comparison



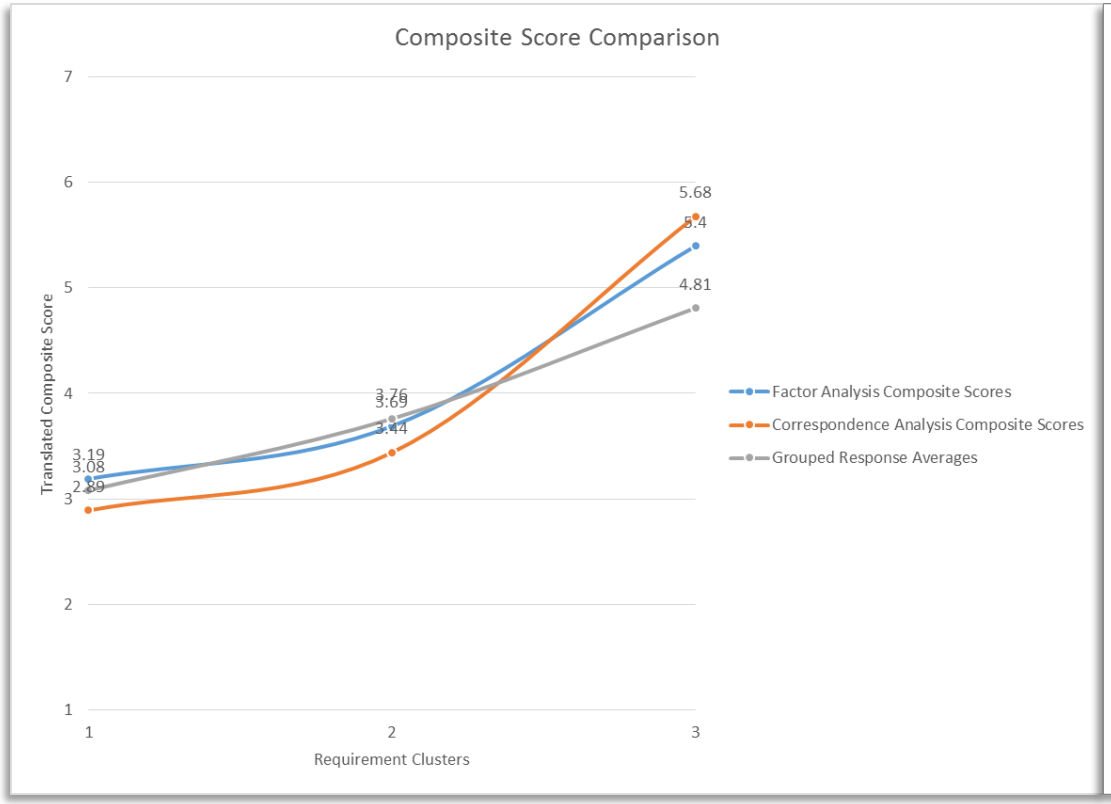
Correspondence Analysis Composite Scores are the mean CA score of clustered requirements. Interpretability is limited due to the clusters underlying Factor Analysis.

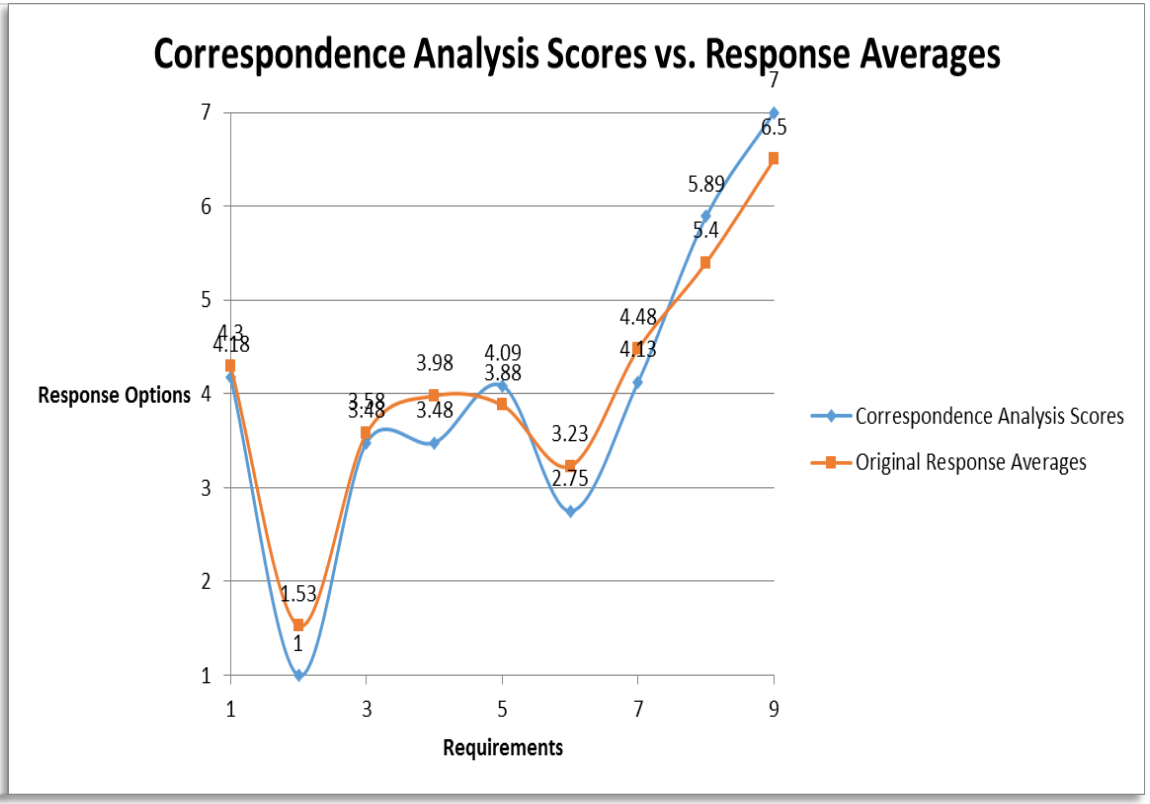
Correspondence Analysis and Averages Comparison



Mean difference of 0.37, max difference of 0.53.

The Propriety of Averaging ?



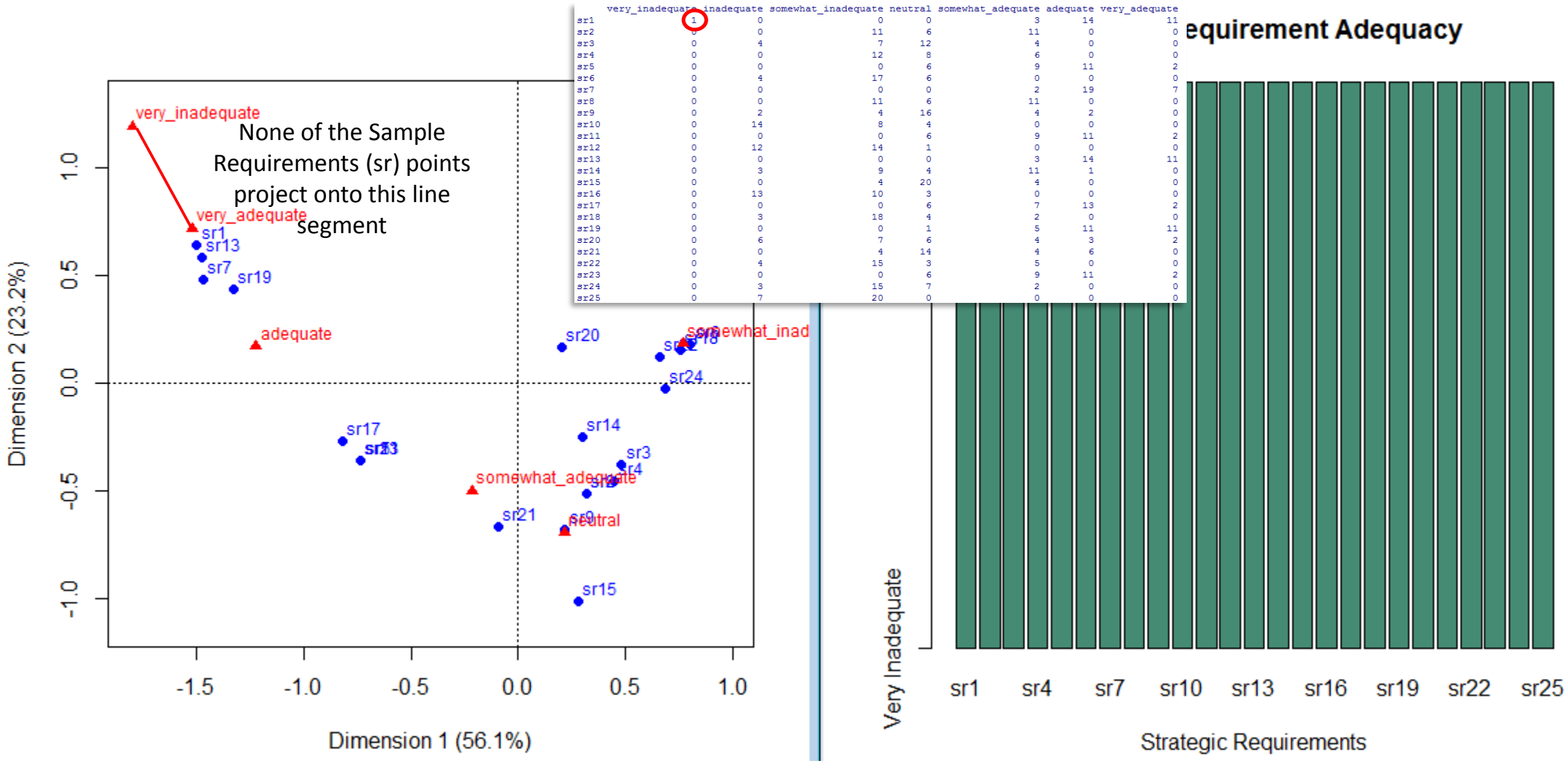


"…measurement, training and research Lite are not good enough or acceptable anymore, and that the practice of analyzing 'Likert scale' questions item-by-item and presenting the results the same way, and as an unorganized laundry list and fuzzy jumble (whether done quantitatively or qualitatively) must simply stop as a research and reporting practice; and that the various persistent myths and urban legends about Likert scales and Likert response formats must be eliminated once and for all through better education and training."

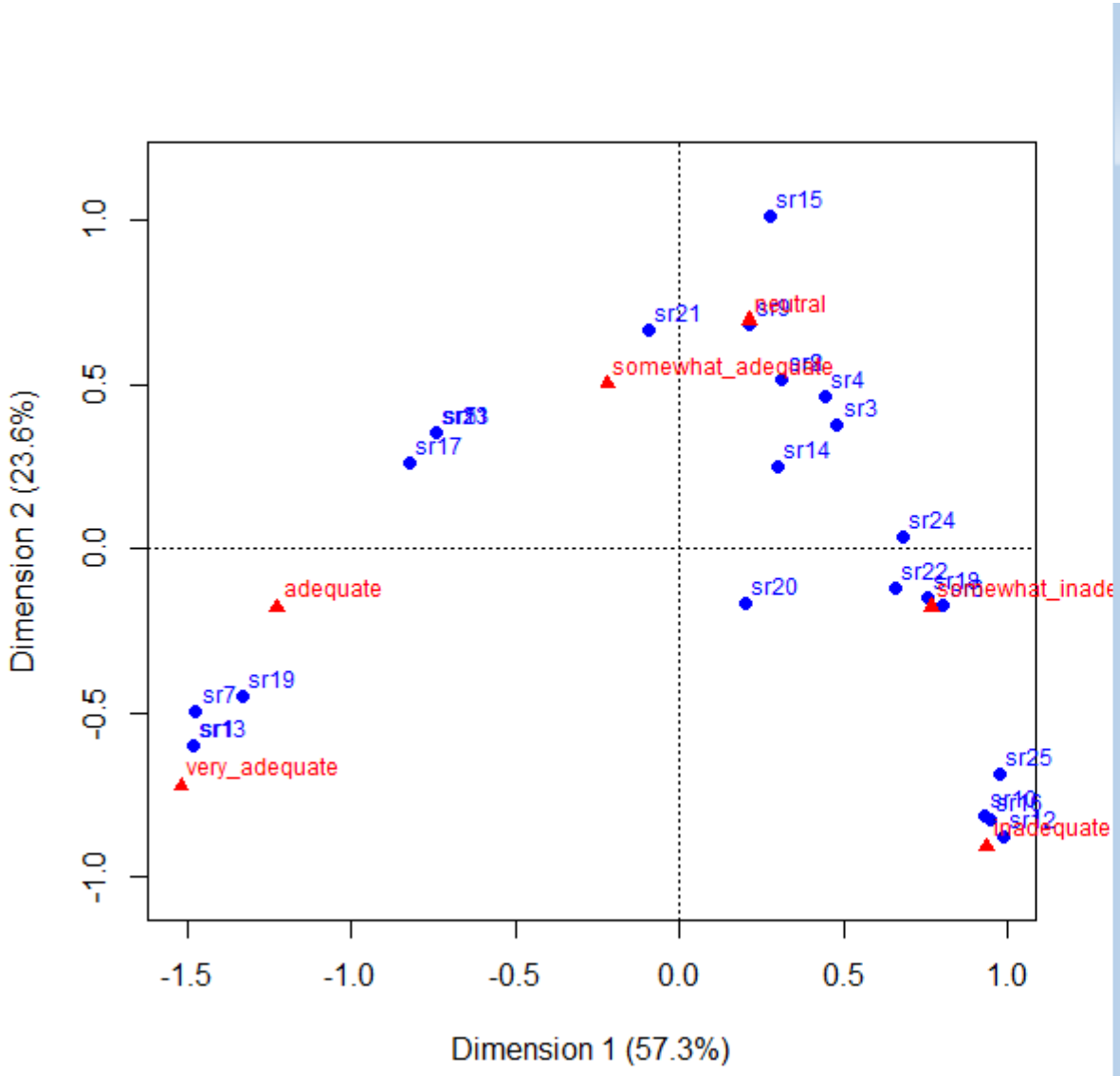
- Carifio and Perla (2007)

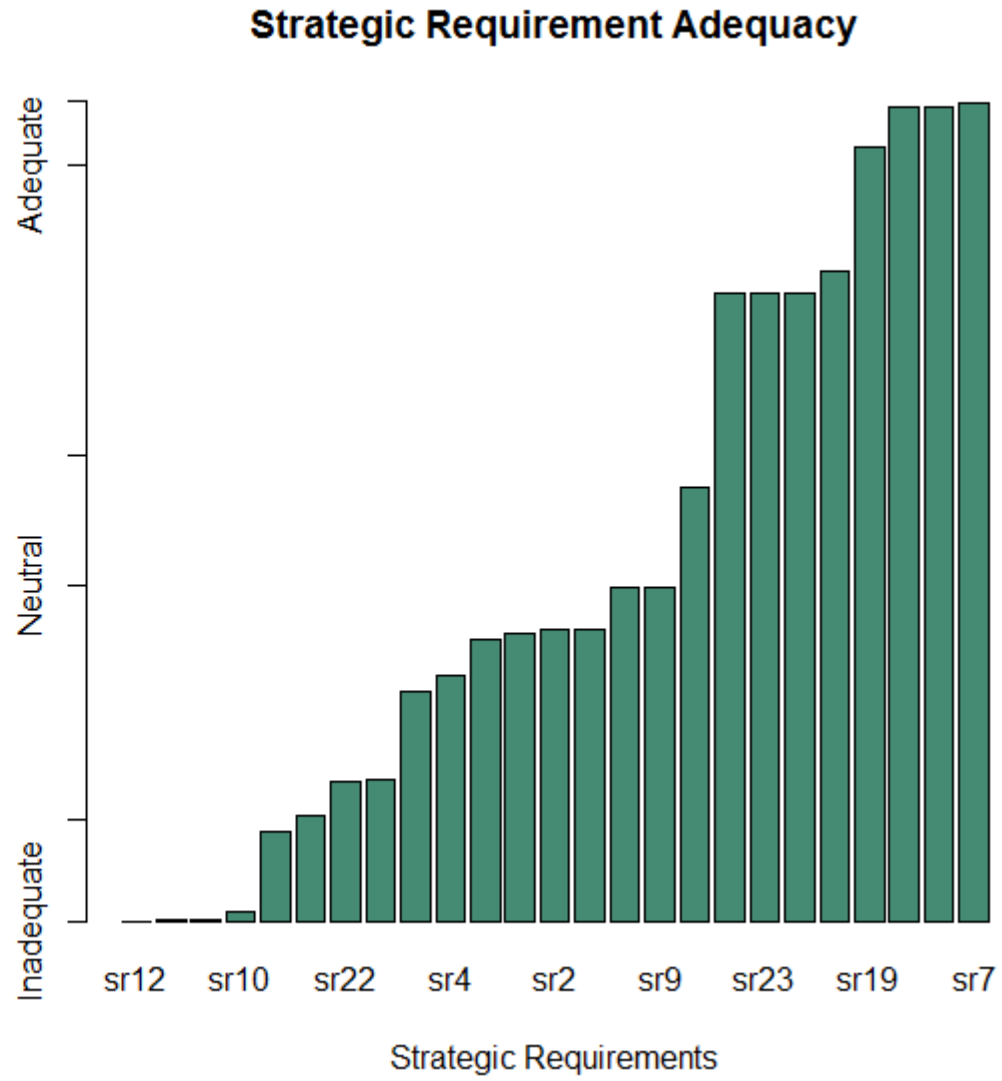
Data Structure: Breaking the Model

	very_inadequate	inadequate	somewhat_inadequate	neutral	somewhat_adequate	adequate	very_adequate
sr1	1	0	0	0	3	14	11
sr2	0	0	11	6	11	0	0
sr3	0	4	7	12	4	0	0
sr4	0	0	12	8	6	0	0
sr5	0	0	0	6	9	11	2
sr6	0	4	17	6	0	0	0
sr7	0	0	0	0	2	19	7
sr8	0	0	11	6	11	0	0
sr9	0	2	4	16	4	2	0
sr10	0	14	8	4	0	0	0
sr11	0	0	0	6	9	11	2
sr12	0	12	14	1	0	0	0
sr13	0	0	0	0	3	14	11
sr14	0	3	9	4	11	1	0
sr15	0	0	4	20	4	0	0
sr16	0	13	10	3	0	0	0
sr17	0	0	0	6	7	13	2
sr18	0	3	18	4	2	0	0
sr19	0	0	0	1	5	11	11
sr20	0	6	7	6	4	3	2
sr21	0	0	4	14	4	6	0
sr22	0	4	15	3	5	0	0
sr23	0	0	0	6	9	11	2
sr24	0	3	15	7	2	0	0
sr25	0	7	20	0	0	0	0

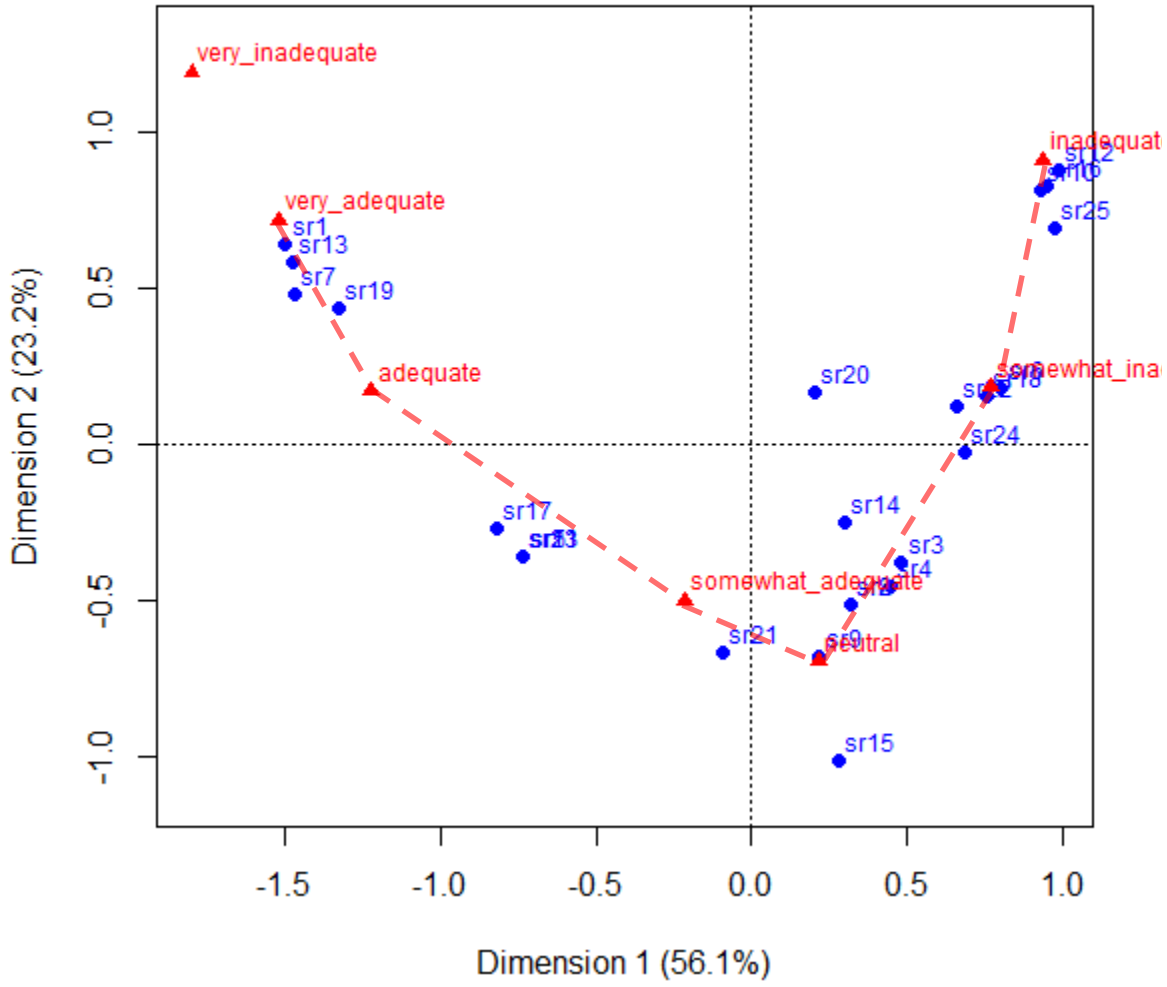


Remove the Offending Column: Sensitivity





Point to Point Projection



- Similarly removes specific response option
- Determine appropriate vector
  - Averages?
- Distances between Likert anchor points directly interpretable
- Difficult to automate
- Higher fidelity distance calculation

Advantages/Disadvantages

Factor Analysis Composite Scores

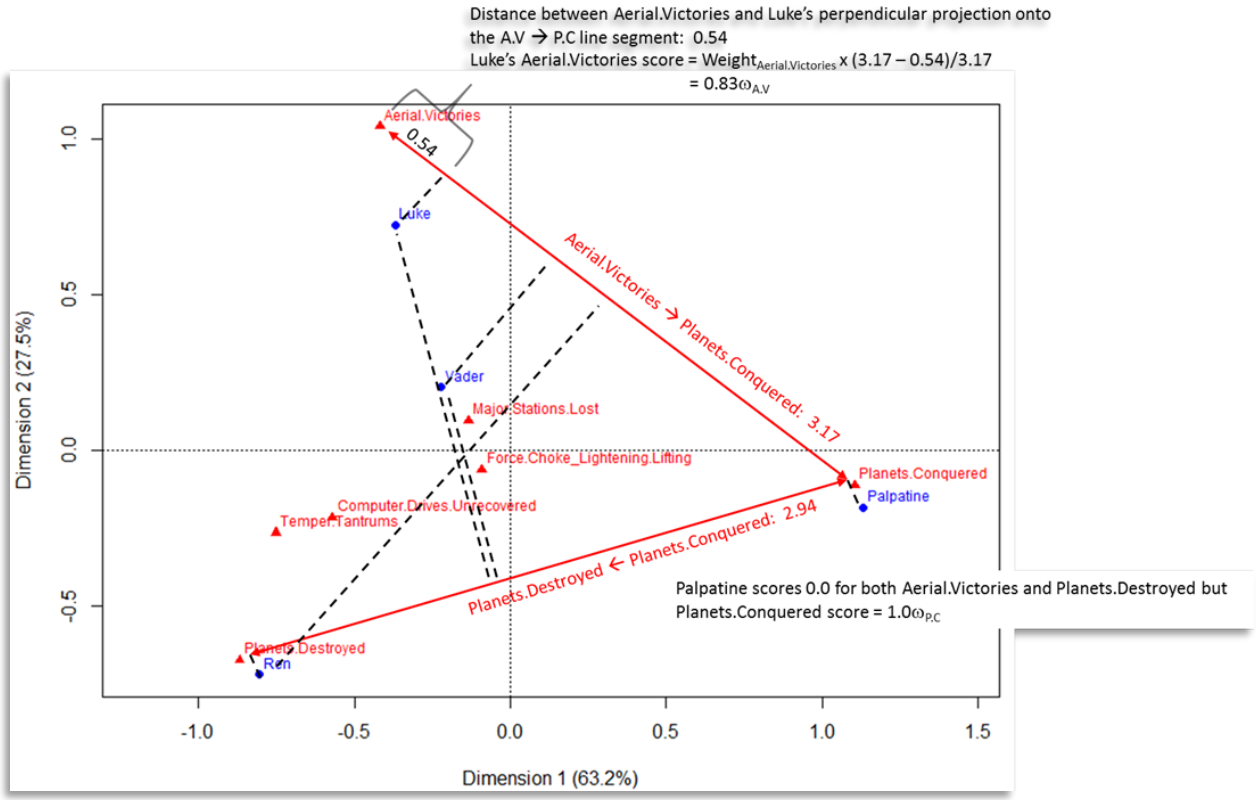
- Established methodology
  - Starkweather (2012)
- Latent factor emergence
- Requires clustering
- Random variable

Correspondence Analysis Indicator Scores

- Innovation based on precedence
- Approximation
  - Assigning extreme values
- Question-by-question
  - Constrains analysis plan
- Single score

How does this analysis help the Alliance think differently?

- Translate survey-level (or ordinal) data into scores
  - Easily interpretable
  - Further analysis
  - Quantify subjectivity
- Applicable to open source data
  - World Values Survey
  - Indices
  - Event/violence data
- Olive branch to settle the controversy surrounding Likert data treatment.

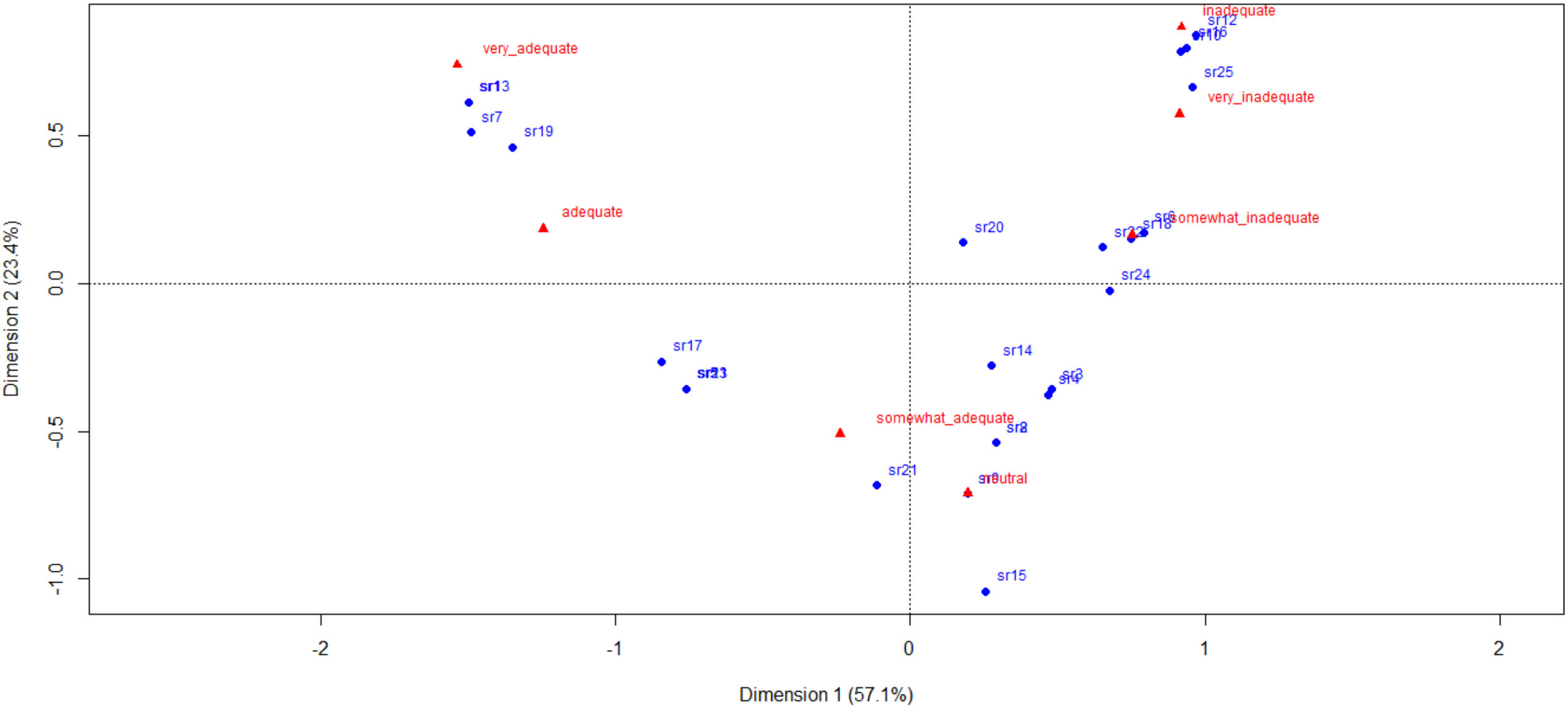


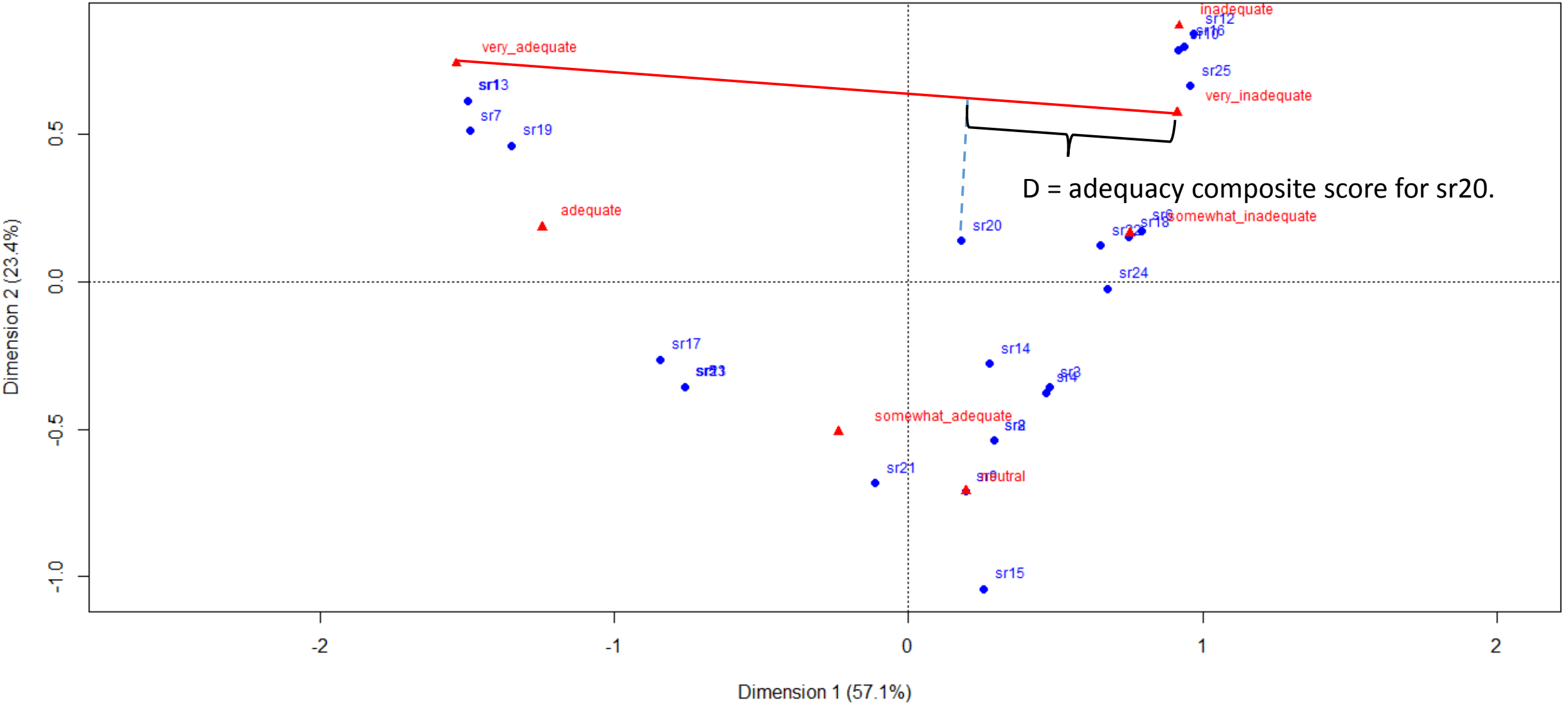
Correspondence Analysis "evil" indicator scores of Darth Vader, Kylo ren, Luke Skywalker, and Emperor Palpatine using violent event data provided by Schramm (2016). http://analytics-magazine.org/five-minute-analyst-dark-side-envelopment-analysis/

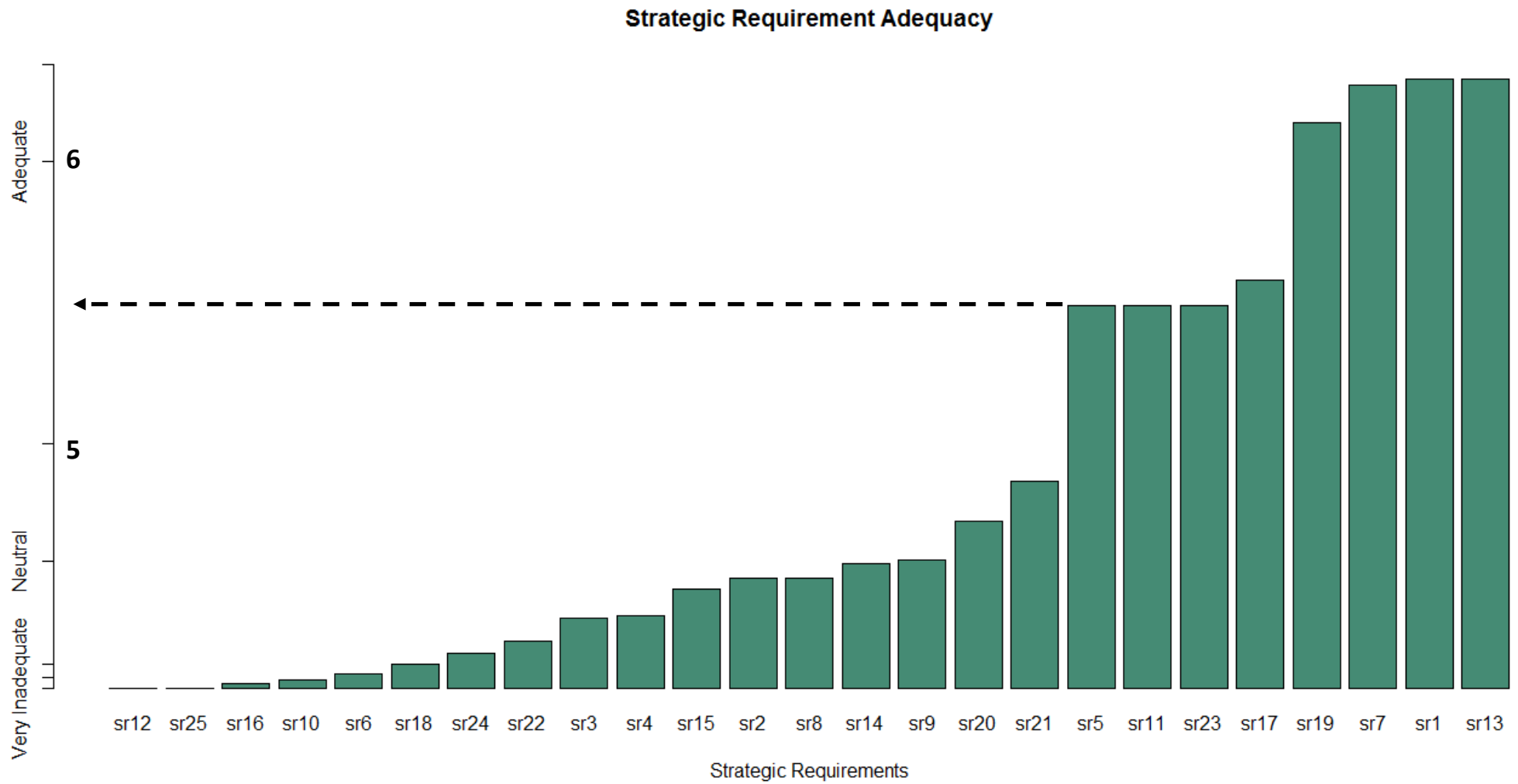
Questions?

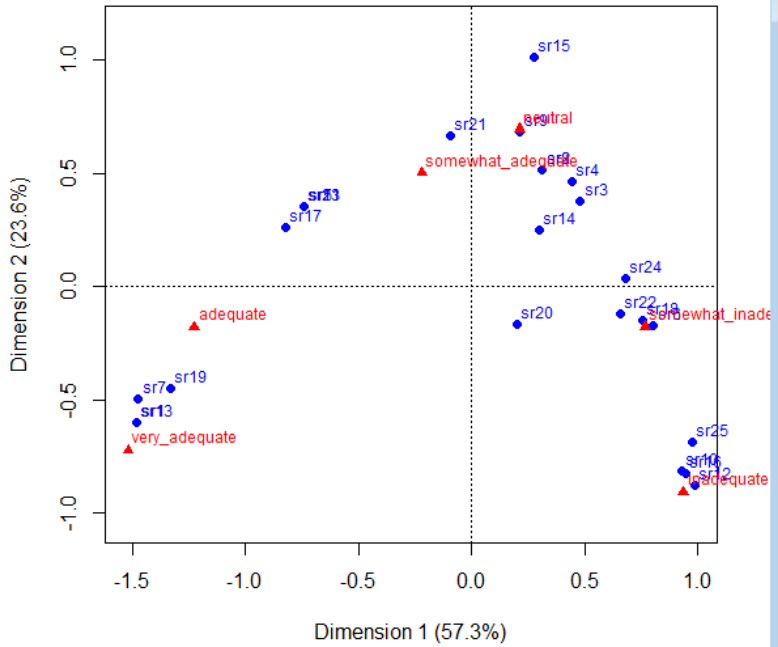
Backup Slides

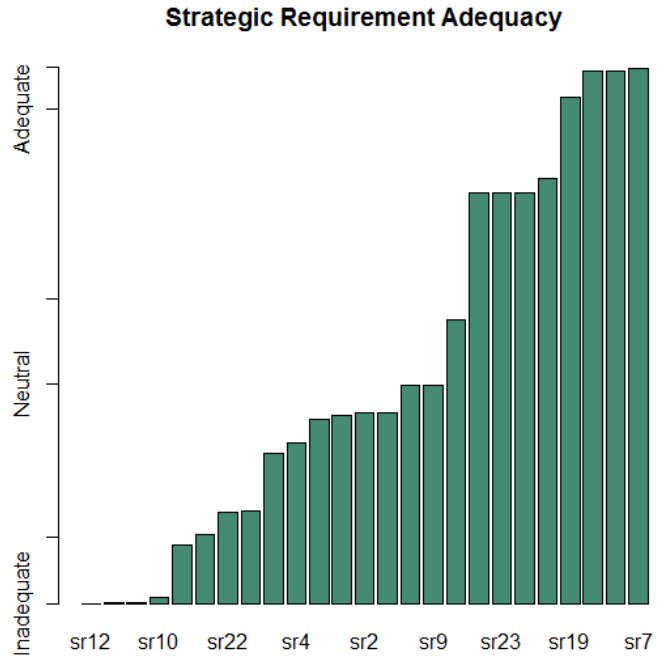
	very_inadequate	inadequate	somewhat_inadequate	neutral	somewhat_adequate	adequate	very_adequate
sr1	0	0	0	0	3	14	11
sr2	0	0	11	6	11	0	0
sr3	1	4	7	12	4	0	0
sr4	2	0	12	8	6	0	0
sr5	0	0	0	6	9	11	2
sr6	1	4	17	6	0	0	0
sr7	0	0	0	0	2	19	7
sr8	0	0	11	6	11	0	0
sr9	0	2	4	16	4	2	0
sr10	2	14	8	4	0	0	0
sr11	0	0	0	6	9	11	2
sr12	1	12	14	1	0	0	0
sr13	0	0	0	0	3	14	11
sr14	0	3	9	4	11	1	0
sr15	0	0	4	20	4	0	0
sr16	2	13	10	3	0	0	0
sr17	0	0	0	6	7	13	2
sr18	1	3	18	4	2	0	0
sr19	0	0	0	1	5	11	11
sr20	0	6	7	6	4	3	2
sr21	0	0	4	14	4	6	0
sr22	1	4	15	3	5	0	0
sr23	0	0	0	6	9	11	2
sr24	1	3	15	7	2	0	0
sr25	1	7	20	0	0	0	0

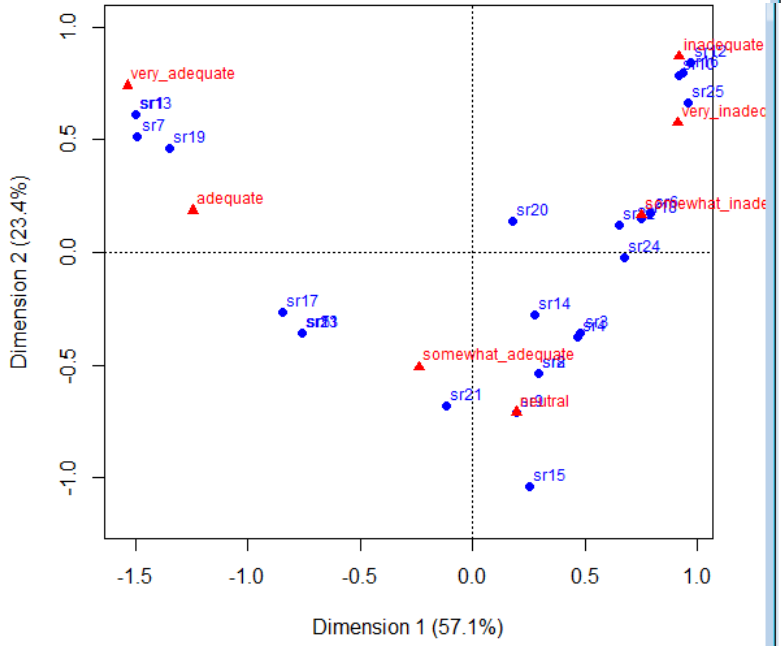

inadequate
sr12
very_adequate
sr25
very_inadequate
sr13
sr7
sr19
adequate
somewhat_inadequate
sr20
sr18
sr24
sr17
sr14
somewhat_adequate
sr21
neutral
sr15
Dimension 2 (23.4%)
Dimension 1 (57.1%)
0.5
0.0
-0.5
-1.0
-2
-1
0
1
2

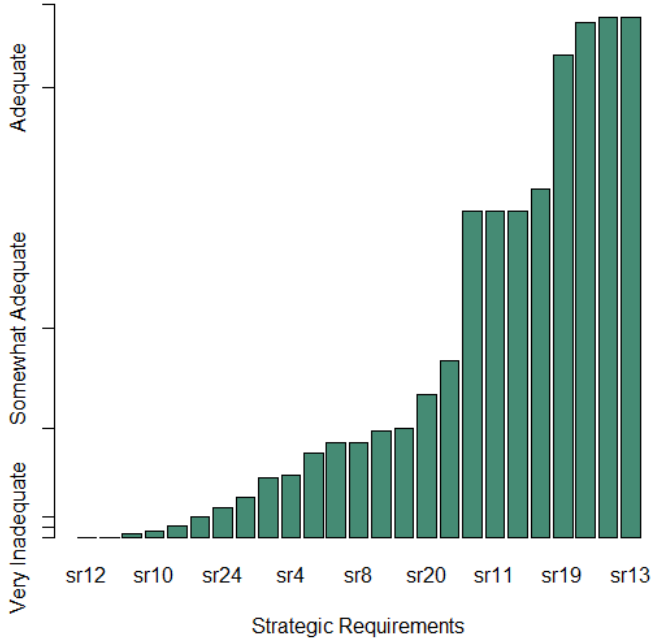

inadequate
sr12
very_adequate
sr25
very_inadequate
sr13
sr7
sr19
adequate
D = adequacy composite score for sr20.
somewhat_inadequate
sr20
sr18
sr24
sr17
sr14
somewhat_adequate
sr21
neutral
sr15
0.5
0.0
-0.5
-1.0
-2
-1
0
1
2
Dimension 2 (23.4%)
Dimension 1 (57.1%)


Strategic Requirement Adequacy
Adequate
6
5
Neutral
Very Inadequate
sr12
sr25
sr16
sr10
sr6
sr18
sr24
sr22
sr3
sr4
sr15
sr2
sr8
sr14
sr9
sr20
sr21
sr5
sr11
sr23
sr17
sr19
sr7
sr1
sr13
Strategic Requirements









With column removed, measurements are nearly identical, Y-axis scale changed

R code for Factor Analysis Composite Scores

```
# From "How to Calculate Empirically Derived Composite or Indicator Scores."
# Dr. Jon Starkweather, Research and Statistical Support Consultant
# http://www.unt.edu/rss/class/Jon/Benchmarks/CompositeScores_JDS_Feb2012.pdf
# Accessed 9 June 2016

# INSPECT LINEARITY OF YOUR DATA --> Assumption of Factor Analysis

# "weighted.sd" takes the vector of values (x) and the weights (w), which are
# the loadings here, and returns the weighted standard deviation of the
# vector of values.
weighted.sd = function(x, w){
                        sum.w <- sum(w)
                        sum.w2 <- sum(w^2)
                        mean.w <- sum(x * w) / sum(w)
                        x.sd.w <- sqrt((sum.w / (sum.w^2 - sum.w2)) * sum(w * (x - mean.w)^2))
                        return(x.sd.w)
                        }

# "re.scale" puts the scores back into the metric of the original questions.
re.scale <- function(f.scores, raw.data, loadings){
                        fz.scores <- (f.scores + mean(f.scores))/(sd(f.scores))
                        means <- apply(raw.data, 1, weighted.mean, w = loadings)
                        sds <- apply(raw.data, 1, weighted.sd, w = loadings)
                        grand.mean <- mean(means)
                        grand.sd <- mean(sds)
                        final.scores <- ((fz.scores * grand.sd) + grand.mean)
                        return(final.scores)
                        }

get.scores.fun <- function(data){
                        fact <- factanal(data, factors = 1, scores = "regression")
                        f.scores <- fact$scores[,1]
                        f.loads <- abs(fact$loadings[,1])
                        rescaled.scores <- re.scale(f.scores, data, f.loads)
                        output.list <- list(rescaled.scores, f.loads)
                        names(output.list) <- c("rescaled.scores", "factor.loadings")
                        return(output.list)
                        }
```

References

1. Starkweather, Jon. "How to Calculate Empirically Derived Composite or Indicator Scores." *Benchmarks Online*, 9 June 2016, http://it.unt.edu/sites/default/files/benchmarks-02-2012.pdf (February 2012).

2. Fricker Jr, Ronald D., Jeffrey A. Appleget, and Walter W. Kulzy. "From Data to Information: Using Factor Analysis with Survey Data." *Phalanx, 30-34* (December 2012).

3. Borg, Ingwer, and Patrick JF Groenen. *Modern Multidimensional Scaling: Theory and Applications*. Springer Science & Business Media. (2005).

4. Carifio, James, and Rocco J. Perla. "Ten Common Misunderstandings, Misconceptions, Persistent Myths and Urban Legends about Likert Scales and Likert Response Formats and their Antidotes." *Journal of Social Sciences* 3, no. 3 (2007): 106-116.

5. Schramm, Harrison. "Five-Minute Analyst: Dark Side Envelopment Analysis." *Analytics*. INFORMS Institute for Operations Research and the Management Sciences. http://analytics-magazine.org/five-minute-analyst-dark-side-envelopment-analysis/ (July/August 2016).